Daily Question – Pure Mathematics - Day 7

Topic: Trigonometry asinΘ±bcosΘ

Day 7 Question 1

(a) Write $\cos\theta - 8\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha \leqslant 90°$. Give the exact value of R and give the value of α to 2 decimal places.

(3)

The temperature of a cellar is modelled by the equation

$$f(t) = 13 + \frac{\cos(15t)° - 8\sin(15t)°}{10} \qquad 0 \leqslant t < 24$$

where $f(t)$ is the temperature of the cellar in degrees Celsius and t is the time measured in hours after midnight.

Find, according to the model,

(b) the maximum temperature of the cellar, giving your answer to 2 decimal places

(2)

(c) the times, after midnight, when the temperature of the cellar is 12.5 °C

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

Day 7 Questions 2

(a) Express $\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α, in radians, to 3 decimal places.

(3)

$$M(\theta) = 40 + (3\sin\theta - 6\cos\theta)^2$$

(b) Find

(i) the maximum value of $M(\theta)$,

(ii) the smallest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $M(\theta)$ occurs.

(3)

$$N(\theta) = \frac{30}{5 + 2(\sin 2\theta - 2\cos 2\theta)^2}$$

(c) Find

(i) the maximum value of $N(\theta)$,

(ii) the largest value of θ, in the range $0 < \theta \leqslant 2\pi$, at which the maximum value of $N(\theta)$ occurs.

(3)

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)